American Sociological Association

American Sociological Review
2021, Vol. 86(4) 759–786

DOI:10.1177/00031224211027893
journals.sagepub.com/home/asr
SAGE

Networks, Property, and the Division of Labor

Emily Erikson[a] and Hirokazu Shirado[b]

Abstract

We use a simulation-based method to consider the effect of different network structures on the propensity for economic producers to develop a complementary division of labor. We use a graph-coloring game, in which nodes are given incentives to find a color that does not match their nearest neighbors, to represent the interdependent coordination problems inherent to the division of labor. We find that a decentralized development of a division of labor is difficult, particularly when too many specializations are chosen. Counterintuitively, a division of labor is more likely to evolve when the ability of agents to specialize is more constrained. The ability to store property also facilitates the development of a division of labor.

Keywords

network science, computational models, markets, social coordination, theory

The division of labor in society is a classic problem of social coordination. In *The Wealth of Nations* ([1776] 2003), Smith attempted to convince government actors to allow the free flow of trade to realize the benefits of an international division of labor. Where Smith saw the development of a division of labor between nations largely as a regulatory problem involving taxes and tariffs, a century later Durkheim considered the possibility of social barriers to the emergence of specialization. In *The Division of Labor in Society* (1893), Durkheim posed a central puzzle of sociological theory: how does complementary specialization—the cooperative interdependence of organic solidarity—emerge from the generalist communities he described as characterized by mechanical solidarity?

Durkheim's answer, although difficult to reduce to a series of hypotheses (Gibbs 2003), contained a formalist structural element. Increasing density of interaction was one precondition for the transition to the cooperative interdependence of the division of labor (Durkheim [1893] 1996:201–223). In contrast, Tönnies ([1887] 2002) described the increasing specialization, industrialization, and marketization of society as the transition from *gemeinschaft*, a society based on dense personal and familial bonds, to *gesellschaft*, a society based on less-cohesive contractual relations. In structural terms, this corresponds roughly to a shift from bonding relations, which provide strength through cohesion and close and redundant relations (Coleman 1994), to bridging or weak ties, that is, more flexible arm's-length relations that provide access to new and different information and resources (Burt 2004; Granovetter 1973). Thus, classical theory gave rise to different

[a]Yale University
[b]Carnegie Mellon University

Corresponding Author:
Emily Erikson, Yale University, 493 College Street, New Haven, CT 06520
Email: emily.erikson@yale.edu

hypotheses regarding the optimal structure of relations for encouraging the division of labor.

The problem of how to achieve a division of labor is not confined to history or theory. Interdependent coordination of complementary areas of specialization and expertise is an ongoing process. Incomplete specialization within nations has been tied to wage stagnation and low rates of economic growth (Rodríguez-Clare 1996). Regional processes of intensifying specialization continue to unfold in the face of vast new infrastructure projects, such as the Inter-Oceanic Highway (Perz et al. 2013) and China's Belt and Road initiative (Lu et al. 2018). Pharmacological specialization among nations has led to a complex web of interdependent trade in health supplies between nations. And as task complexity and ambiguity increases within organizations, self-selection of roles, expertise, and job duties has become increasingly prevalent, making businesses and firms another area in which the process unfolds on a regular basis (Raveendran, Silvestri, and Gulati 2020).

The process of a decentralized development of the division of labor has been documented across several arenas, but the hypotheses raised by classical authors in sociology about the effect of network structure remain to be tested, developed, and explored. The field of economic sociology provides ample evidence that exchange is constrained by preexisting patterns of interaction, whether these are determined by proximity, transportation infrastructure, river networks, or affectual bonds of trust, loyalty, or homophily. We consider here how these durable patterns can affect the development of complementarity that lies at the heart of profitable exchange and market expansion.

To investigate these issues, we conduct agent-based simulations (Macy and Willer 2002) based on the graph-coloring paradigm (Jensen and Toft 2011) that model and analyze the conditions under which complementary specialization may or may not emerge within a networked population of generalist producers. In doing so, we consider the different conditions that inhibit or encourage this transition (Hofbauer and Sigmund 1998; Smith 1982). The graph-coloring problem is a multipartite network-based extension of the four-color problem posed by Francis Guthrie in the late nineteenth century. Guthrie posited that any geographic map of regions would require at least four colors to ensure contiguous regions had different coloration from each other. The puzzle was later generalized to a graph-theoretical context in which the central problem was to identify how many colors were necessary to ensure each node of a network could be colored differently from its directly-connected neighbors. Such networks are multipartite because nodes belong to different sets (or parts) defined by the different colors. The solution to this puzzle was defined by the chromatic polynomial, a function that linked the number of colors used for coloring the nodes to a corresponding number of possible solutions. Graph-coloring is a well-established field of research in graph theory (Jensen and Toft 2011).

Graph-coloring provides means to examine collective action as well as computational challenges. The multipartite networks in the graph-coloring game have been used to explore a number of real-world problems, including scheduling class times, choosing differentiated ring tones, selecting a frequency in broadcasting systems, and choosing an area of expertise to cultivate within an organization (Kearns, Suri, and Montfort 2006; Shirado and Christakis 2017). The game represents in a general and abstract way the larger class of social situations in which individuals engage collectively in distributed problem-solving with restricted, local information. In this sense, the game addresses in a slightly different way the same theoretical problems that are debated in the "collective intelligence" literature and that underlie many assumptions about market properties: how can decentralized individuals collectively resolve difficult social-coordination problems?

Such models are abstract, but a robust tradition in sociology uses formal models to produce valuable insights into social phenomena

(Boorman 1974; Bruch and Mare 2006; Centola and Macy 2007; Feld 1981; Granovetter 1978; Kitts 2006, Page 2008; Watts 1999; White 1963). Using computational models that vary and compare different networks of agents attempting to coordinate, we find that network structure does indeed have an effect on the likelihood and rate of complementary specialization, although in a different way than implied by Durkheim or Tönnies. In particular, the chromatic polynomial for a particular network has a strong effect on the number of agents who engage in a division of labor. However, that effect works in an unexpected direction. Previous research shows that agents more easily coordinate when there are more solutions to the graph-coloring problem within their network (Shirado and Christakis 2017). These findings make intuitive sense, as we would expect that increasing the number of paths through a maze should make solving it easier. However, we find that networks with many chromatic solutions inhibit coordination between agents. Networks with few chromatic solutions (i.e., with many selection restrictions in the solution space), in contrast, appear to provide a structural guide that facilitates the development and expansion of a division of labor. In our models, complementary specialization is also strongly affected by the capacity to store surplus goods, which we understand through the lens of property rights. We find that the ability to store property has a strong positive effect on the likelihood that networks will achieve a high proportion of agents able to successfully divide their labor with others. And different network topographies also appear to be more and less suitable for encouraging a division of labor among agents.

The findings suggest a revision of our understanding of the conditions that foster the decentralized development of the division of labor; as such, they address long-standing questions about the need for centralized or state intervention to achieve certain types of economic coordination, the conditions under which such centralized intervention may be most necessary, and the role of private property regimes in economic development. Our results suggest simple heuristics that may help foster successful divisions of labor. They also offer interesting suggestions as to why two-part divisions of economic activity, such as hunting and gathering or public and private, are so pervasive in social organization.

Our study supports previous work in showing that network structure plays an important role in determining the extent of the development of complementary specialization; however, the structural features previous theorists focused on—bonding and bridging ties—may be ancillary to the transition. Instead, the minimum number of colors needed to solve graph-coloring and the number of solutions (i.e., the number of possible color combinations with the minimum number of colors), which vary greatly over different network structures, appear to serve as a previously hidden substratum constraint on the emergence of specialization in the formal models.

THE DIVISION OF LABOR PROBLEM

Adam Smith has long been recognized as the preeminent theorist of the division of labor. Smith, however, was not as concerned with the problem of how specialization develops over time as with promoting its benefits. Smith assumed that knowledge of the benefits of the division of labor would be enough to encourage specialization. For Smith, and for many who followed him, the limits of specialization were set by the size of the market (Becker and Murphy 1992).

By the nineteenth century, social observers were finding signs that market expansion and increasing specialization were accompanied by other fundamental changes in social structure. As noted earlier, Durkheim suggested that an increasing density of interactions played an important role in his theory of the change from homogenous, independent producers to interdependent specialists. Tönnies ([1887] 2002) famously characterized the transformation as one from *gemeinschaft* to *gesellschaft*, in which small homogenous communities bound by personal ties based in natural will became larger, more complex societies bound by contractual and impersonal ties based in rational will. Polanyi (1944) made similar arguments about the loss of community ties in market society. The influence of these three thinkers was enough to turn the decreasing density of community ties into a major narrative of the transition into modernity, industrialization, and capitalism.

In the twentieth century, much of the sociological work on the division of labor shifted away from the idea of a historical transformation into a specialized market economy and instead explored how the system of roles and occupations differentially distributed effort and rewards across the population (Abbott 1988; Strauss 1985). Hechter (1978), for example, documented the positive effect of occupational specialization within ethnic groups on group solidarity. A large stream of research continues to measure the inequities between men and women in household work (i.e., the domestic division of labor) (Shelton and John 1996), and recently, research has documented the effect of the division of labor in creating earnings inequalities within organizations (Wilmers 2020). We differ from these works in focusing not on the existence or effect of the division of labor, but instead on the conditions that foster its development.

Organizational ecology and resource-partitioning theory has shed significant light on environmental conditions that encourage specialization and generalization (Carroll 1995; Hannan and Freeman 1977). These theories are based on evolutionary models, so rates of specialization and generalization are not determined by organizational choices as much as the survival rate—or fitness of organizations—in particular environmental conditions. In niche theory, the rate of environmental change is a large factor in specialization and generalization processes: stability encourages specialization, and instability encourages generalization, which is more robust across various conditions. In resource-partitioning theory, returns to scale encourages generalization. Other central factors determining the adoption of specialization and generalization are the availability of resources (fundamental to both), the prevalence of competition between organizations within a niche or for the same resources, and the degree of centralization that occurs in a market (Carroll 1995; Hannan and Freeman 1977; Hannan, Pólos, and Carroll 2007). The network structure of exchange between these organizations is not of primary importance for good reason: the model of relations between firms is one of competition rather than interdependence.

The difference between competition and interdependence highlights a scope condition of our research, which cannot address the full range of specialization processes. The division of labor cannot take place without specialization, but the processes can be distinct. We use the phrase "the division of labor" to refer to a process in which the labor required to produce a good or satisfy a need is divided between parties. This process has also been called the arc of work, project arc, or task articulation (Strauss 1985). In Smith's famous example of a pin factory, one person can make a pin, but the division of labor differentiates specialized tasks within the larger process of making a pin and assigns them to different individuals, thereby increasing overall productivity. If all the individuals do not continue to make their different parts, no pin will be produced. This example fits our definition of a division of labor in which tasks are interdependent and complementary. Specialization, however, can take place without interdependency. For example, Carroll's now classic work, *Publish and Perish* (1987), considers generalization and specialization within the newspaper industry. Newspapers do not depend on each other for their existence, and they do not divide a common task. There is, however, complementarity across industries: newspapers could not exist without a differentiated market economy that supplies other necessary goods for exchange (e.g., paper products and printing machinery). This interdependency, however, is not the focus of resource-partitioning theory.

Our research is instead relevant to work focused on the organizational and social determinants of *complementary* specialization, which may be further distinguished from interdependent specialization based on difference. Specialization based on difference requires actors to distinguish themselves from others, but specialization based on complementarity can tolerate difference as long as the complementary parts, goods, or portions of the required task are available to agents. An example of successful specialization based in difference is finding a unique ring tone, whereas complementary specialization is closer to the coordination of a four-part harmony, in which some parts may be doubled but all the parts are necessary for the music to emerge as intended.

Research on complementary specialization has had most purchase within the study of complex organizations. A large amount of research has been devoted to task articulation and differentiation within organizations (Lawrence and Lorsch 1967; Perrow 1986; Simon and March 1958; Thompson 1967; Weick 1969). Whereas much of this literature examines the managerial imposition of specialization based on task characteristics through cultural integration, goal alignment, and other strategies, we instead investigate bottom-up processes of differentiation relevant to an emerging stream of research on self-organization in teams and organizations and the self-selection of tasks (Raveendran et al. 2020).

Various authors argue that employees have always had a hand in autonomously creating their own organizational roles through job-crafting (Raveendran et al. 2020; Wrzesniewski and Dutton 2001), sculpting (Bell and Staw 1989), or task-bundling (Cohen 2013). Cohen (2013), for example, documented the installation of a new DNA-sequencer in nine different laboratories, finding that in each case, with the introduction of the new technology, the new tasks were allocated in a decentralized, iterative process based in employees trying out the equipment, seeking knowledge, interpreting results, and reconciling different needs. Puranam, Alexy, and Reitzig (2014) show that decentralization of these decisions in organizations may be increasing. Similar processes are at stake in Starks's (2011) idea of heterarchies, in which employees leverage interactions with colleagues who have different areas of expertise and knowledge to improve their analytic power, problem-solving abilities, and innovative capacity.

These works do a wonderful job of digging into the relational processes involved in decentralized task and knowledge articulation, but they do not formally address the effect of network structure. Outside of organizations, work in economics has incorporated the idea of coordination costs as an additional complicating factor affecting the development and expansion of specialization in markets (Becker and Murphy 1992), and heterodox economists have considered the role of social interaction in production decisions (Foley 2019). However, recent research has done little to mobilize our understanding of network structure and dynamics to consider how variation in network topology can affect the process of achieving complementary specializations within organizations or in the market arena. We take this step here.

FORMALIZING THE DIVISION OF LABOR

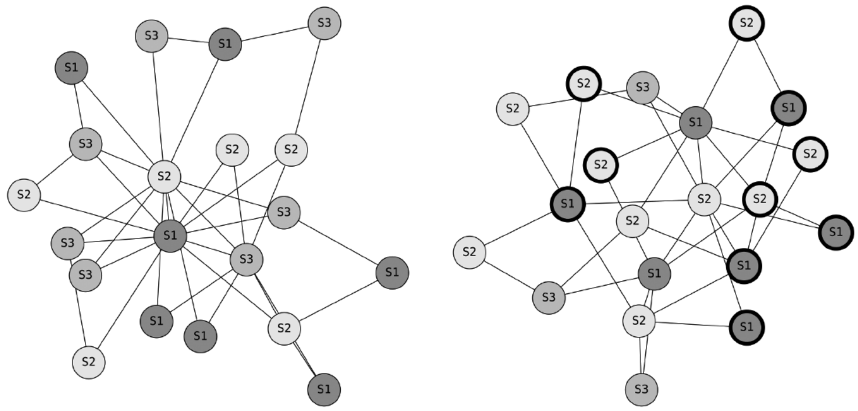
Following Smith, the benefit of the division of labor arises from specialization and differentiation: there is no reason for either a dog or a person to exchange a bone for a bone (Bearman 1997). If everyone specializes in bones, there are no gains to trade. If it is easy to locate partners who produce different goods, people are more likely to specialize; otherwise, they are less likely to specialize (Diamond 1982). It follows that successful, or profitable, specialization is an interdependent choice, where the choice of specialization depends on others' choice of specialization. The graph-coloring game begins with a network of interconnected nodes. Nodes in the network may take on one of a defined set of colors. The goal is for each node to take on a different color than its neighbors (Kearns et al. 2006). In this way, the game effectively captures the fundamental features of interdependent specialization, in which colors indicate specialization categories.

In the graph-coloring game, agents attempt to differentiate themselves from others. We impose a further constraint to better capture the process of the division of labor: the process of dividing the labor required for the completion of a task, the production of a good, or the fulfillment of a need requires complementary specialization. If, for example, we begin with the idea of how the division of labor might emerge in a subsistence economy, we might first assume humans have more than one basic need. This assumption implies that an absence of specialization requires generalization. If we assume humans require water, food, and shelter, then to survive, each individual must work to procure the water, food, and shelter necessary for their existence. All three items are necessary. To specialize in any one good, a person must be able to acquire the other goods from someone else. If one person acquires water, someone else must procure food, and another must make shelter. For complementary specialization to occur, the individuals must coordinate their activity with each other to survive, and the decision process is distributed across actors. To capture this type of coordination, our agents have incentive to specialize in goods that complete the set of goods offered by their near neighbors, and we count a successful division of labor as one in which an agent is able to acquire each of the complementary goods. We call this modified version of the graph-coloring game *the division of labor game*.

Two measures are important to understanding the features of graph-coloring games (Jensen and Toft 2011) and our closely related model of the division of labor. The *chromatic number* of a graph is the minimum number of colors for which a solution to a graph-coloring game exists. In the graph-coloring game, a solution is one in which every node in the graph is a different color than its neighbors. If there are too few colors, no solutions will exist. The structure of the network can alter the minimum number of colors necessary to solve the problem. For example, in a ring network in which each node has two neighbors and there is an even number of nodes, the chromatic number is two. A solution is possible if nodes alternate between colors. A more complex random network of the same size may require more colors for a solution. In general, the chromatic number will vary depending on the number of nodes, the number of ties between nodes, and the topological properties of the network.

The second measure is the *chromatic polynomial*. As noted earlier, the polynomial is a function that relates the chromatic number to the corresponding number of total solutions that exist to the graph-coloring problem in a given network. A total or complete solution is one in which every node in the graph is a different color than its neighbors. A polynomial is necessary because the number of solutions varies with the number of colors used. The number of solutions the chromatic polynomial counts varies depending on features of the network, such as the number of cycles and closed triangles, but a precise measure of the number of solutions cannot be detected using many basic descriptive structural measures. For example, Figure 1 presents two networks generated through simulated processes of preferential attachment (Barabási and Albert 1999), a standard generative model in social network research. The networks were generated through the same random preferential-attachment process. They both have 20 nodes, the same chromatic number, and the same density. However, the chromatic polynomial for the chromatic number of 3 returns 12 patterns of full-coloring solutions for the left network and 8,280 solutions for the right network. Small and difficult to observe changes in network topology produce extreme variance in the size of the set of possible solutions.

Figure 1 also illustrates the difference between the types of specialization present in the original graph-coloring problem and in our division of labor game. In the graph-coloring game, all agents in both networks



Chromatic solution number = 12 Chromatic solution number = 8,280

Figure 1. Two Preferential-Attachment Networks with the Number of Solutions

would be considered to have successfully solved the game because each agent has a different color from all their neighbors. In the division of labor game, the agents (i.e., nodes) marked by the darker outline have *not* solved the puzzle because they do not have neighbors that represent all the different colors. For the complementary specialization required for the division of labor, agents need to be directly linked to neighbors that produce all of the other goods required by the game.

CHROMATIC CONSTRAINT AND PROPERTY

To discuss elements of the chromatic polynomial, we introduce a new term: *chromatic constraint*. The chromatic polynomial links the chromatic number to a corresponding number of solutions. When the number of solutions is high, we consider the chromatic constraint to be low. When the number of solutions is low, we consider the chromatic constraint to be high. In this case, constraint specifically refers to the number of options available to nodes as they choose a specialization. Figure 1 clarifies this point further. In the network with a low solution number (solution number = 12), the color-options available are highly constrained; each node's color is largely determined by the colors of its neighboring nodes. In contrast, the network on the right with a high solution number (solution number = 8,280) has many nodes that are structurally unconstrained (indicated with a bold outline). In the graph-coloring game, these nodes can choose more than one different color based on their direct neighbor's decisions. The solution number increases rapidly with the number of unconstrained nodes, resulting in an extremely high variance in the measure.

This formalization allows us to extend previous work on the structural properties that encourage the emergence of the division of labor. Our expectations are in line with previous work in so far as we expect network structure to matter for the outcome of interest, but we expect the effect of that structure may be further refined by drawing from recent research on social networks and decentralized coordination. Kearns and colleagues (2006) show that experimental subjects are better able to solve cycle graphs than preferential-attachment networks. McCubbins, Paturi, and Weller (2009) found that increased density of connections in several stylized graphs helped experimental subjects find successful solutions. Shirado and Christakis (2017) show that adding noise via preprogrammed bots can help experimental subjects solve the coordination problem. One element that remains in the background of these important studies is that the likelihood of solving the graph-coloring problem is strongly related to the chromatic polynomial.

The chromatic polynomial of a network is important to solving these coordination problems because it changes the solution space. Social coordination—including cultural convention (Mackie 1996), diversity and inclusion (Page 2008), knowledge management (Gomez and Lazer 2019), resource exchange (Diamond 1982), and the division of labor—occurs within a complex solution space containing several suboptimal and optimal states. The nature of coordination varies with the landscape of solutions spaces, which cannot be directly observed or computed (Garey and Johnson 1979).

Consistent with previous research, we find that the chromatic number, which in our case represents the number of specializations, is important in determining whether the division of labor takes hold in a population. We also find that chromatic constraint is an important factor in determining the success of the division of labor. But, in contrast to previous research, we find that chromatic constraint *encourages* the division of labor. In the graph-coloring game, low chromatic constraint (i.e., a high solution number) is associated with a high likelihood of solving the problem (Shirado and Christakis 2017). This outcome makes intuitive sense. In networks with higher constraint, the problem has fewer solutions. It follows that those solutions may be harder to find, and agents can be trapped in ineffective coordination that duplicates efforts by their neighbors. The network is like a maze, and the large number of suboptimal solutions act as dead ends. Higher levels of constraint can make it more difficult for agents to reach a global optimum of social coordination, because the solution options are more limited.

However, under certain conditions, high levels of constraint can guide actors to a solution. On a dark night, it may be easier to navigate a narrow canyon than an open plain. It is a simple decision to specialize in food if you are connected to only two neighbors, one of whom specializes in water and the other in shelter. Ring-lattice networks have high chromatic constraint, and the repeating pattern of ties that run through them can provide a guide that makes it easy for agents to choose a specialization that will solve the complementarity problem. In these cases, high levels of constraint can lead to a larger proportion of complementary specialists, as decentralized decision-makers follow a narrow, repetitive pattern. Instead of trapping agents, the structure acts as a guide. Somewhat counterintuitively, if agents have more latitude to adopt different colors, there may be no clear choice. In these cases, low chromatic constraint can impede the spread of specialization across nodes. Notably, tightly constrained patterns of exchange are found in many noncapitalist exchange systems, such as the Kula Ring of the Trobriand Islands (Malinowski 1984; Schieffelin 1981; Strathern 1971).

We also expect agents' capacity to store property will facilitate the division of labor, especially in networks with low chromatic constraint (i.e., few chromatic solutions). Storage capacity may be thought of as good-specific (e.g., fish is more difficult to preserve than grain), technological (the invention of refrigeration vastly expanded societies' capacity to store food products), or surplus wealth that allows for innovation (e.g., funding for research labs in for-profit firms). In all cases, however, storage implies recognition of the idea of property protections for individuals.

Scholars have defined property as the right to use a good, the right to exclude others from use of that good, and the right to transfer those rights to others (Carruthers and Ariovich 2004; Reeve 1986). Property rights vary significantly across societies. Property regimes in western societies are often characterized by rights enforced for the individual (i.e., private property regimes) (Ostrom 1997). However, even in European or European-derived societies, there are numerous instances of collective property and pooled resources shared across communities, such as the commons or fishing rights, and open-access regimes. Many First

Nations in the Americas likely had common property and open-access regimes that nullified the right to excludability (Carruthers and Ariovich 2004; Graeber 2011; Reeve 1986). Some contemporary societies, such as the Toba of Argentina, have egalitarian norms that preclude withholding goods or resources from others (Kapsalakis 2011).

In our model, the idea of private property, understood as an agent's capacity to store surplus goods, allows actors to fulfill the needs they require and simultaneously set aside a specialized good for potential future exchange. We assume that without private property rights, agents would not be able to preserve their property from others. This capacity does not exist (or has a tenuous existence) in societies without formal or normative protections for private property. Without the capacity to store goods, agents have to exchange goods simultaneously with production (i.e., in the same round). Generalists have to wait for their partners to choose a specialization to be sure they choose a complementary specialization. If agents do not have a clear choice, they will not choose to specialize, as they cannot be sure they will obtain the goods they require or desire through exchange with others. Each agent has to find a local solution to the complementarity problem in order to specialize, and the complementarity problem has to be solved in a deterministic way.

When agents can store property, they may specialize in a good before other exchange partners have chosen their specialization, effectively putting aside a specialized commodity for exchange with a partner who may choose a different specialization in future rounds. When storage is possible, the solution to the specialization problem can be found stochastically through the rearrangement of successive specialization patterns. Agents who have found different local solutions to the complementarity problem can readjust if their solutions are not globally optimal. They can continue to experiment, which allows a larger number of agents to find successful patterns of complementary specialization.

SIMULATING EXCHANGE USING THE DIVISION OF LABOR GAME

To explore the effect of network topology on the successful coordination and achievement of a mutually beneficial division of labor between producers, we conduct agent-based simulations in a fixed multipartite network structure. The nodes cannot reconfigure their ties, because doing so would change the structure of the network, making it impossible to measure network structure as a constant affecting the outcomes of the coordination process. In the simulation, we first generate a network. Within this network, a small cluster of nodes are randomly assigned to an initial state of specialization.[1] That is, we randomly select a dyad in a network and assign different colors to the nodes for 2-color games if the network's chromatic number is 2. We color a triangle for 3-color games, and a 4-node complete subgraph for 4-color games.

We also varied the number of colors assigned to nodes to explore the importance of the number of specializations in the development of the division of labor, and the relevance of the chromatic number. In each variation, we examine the evolution of complementary specialization in a network, that is, how far a minority of agents using complementary specialization can spread the complementary mode of production to the population of generalists (Hofbauer and Sigmund 1998; Smith 1982). We assume all nodes are producers. There are no brokers, because brokerage is itself a kind of specialization. To imitate conditions that encourage the division of labor, we do not impose caps on productive capacity, so nodes may exchange goods with any number of partners.

In previous experimental applications of the graph-coloring game, individuals had external incentives to identify solutions to the game. Participants were given a small sum of money each time they successfully identified a complete solution for a specific network (Kearns et al. 2006; Shirado and Christakis 2017). Applying the game to the problem of the division of labor required reconfiguring the incentives so they more closely model a decentralized exchange process. We adjusted the game by incorporating a payoff structure in which agents have a choice to generalize or specialize but receive a greater benefit from specialization when they can also acquire the other goods they need from exchange (Foley 2019). Agents assigned to each node of a network calculate the payoff with the specializations of their network neighbors to update their own node color (i.e., specialization of an item or generalization).

The payoff structure with two neighbors and three items is represented in Figure 2. Choosing generalization (G) provides a return of 1 regardless of alters' status. Specialization (S1, S2, and S3) gives ego a higher return ($R > 1$), but only when alters specialize in the other items; otherwise ego earns nothing. Agents will specialize as long as their needs are met, so they may be tied to someone specializing in the same good as long as they also have access to nodes that produce the other goods required for the higher payoff. It follows that the repetition of colors among alters does not prevent ego's specialization (e.g., if an ego has three neighbors and they have S1, S1, and S2, the ego will still benefit from choosing S3 in a three-color game).

Figure 3 presents the simulation process beginning at this point and shows how property is incorporated into the simulation as a threshold. The threshold determines how long agents can store goods to earn future specialization benefits. If the threshold is 0, each agent selects specialization only when the payoff for specialization is better than that of generalization under the local conditions in that round.

The simulation begins when one agent is chosen at random to update its state based on the payoff it could receive from its neighbors. When storage is not allowed (i.e., the threshold = 0) or the agent's standby counter reaches the threshold, the agent chooses either a specialization or generalization according to the expected payoff outlined in Figure 2. For example, in a 3-color game, the agent chooses a different specialization from its neighbors when its neighbors have chosen specializations that are different from one another (a color; i.e., S1, S2, or S3). This sequence is represented in pathway A of Figure 3. If an agent's neighbors have not chosen two distinct specializations, the payoff to specialization for the agent does not exceed generalization, and it chooses generalization (white; i.e., G), represented in pathway C. When storage is allowed (i.e., the threshold is greater than 0) and the agent's standby counter is less than the threshold, the agent follows a simple greedy strategy and chooses to store a specialized good that minimizes overlap with the specializations of its neighbors (Chaudhuri, Chung Graham, and Jamall 2008). This sequence is represented in pathway B. After the update, if the agent has put aside specialized goods for future exchange, its standby counter is incremented by one; otherwise the counter is reset to zero. Then, another agent is randomly chosen to update its state. This process is repeated 5,000 times.

We evaluated the simulation outcomes using the fraction of agents who have a full set of complementary specializations with neighbors and therefore benefit from the division of labor at the end of the session. We repeated the process 100 times using different random sequences of agents' decision moments, including the initial color assignment. One observation is the average proportion of these 100 realizations. With the exception of the ring-lattice network (which has no variation in outcomes because of its topology), we tested 500 networks for each network model.

NETWORK TOPOLOGY

Our aim here is to examine how network structure affects the dynamics of the division of labor, with attention to how varying degrees of storage capacity can affect that dynamic. To examine the effect of specific network characteristics in the division of labor while controlling for others, we use two well-theorized and common types of stylized networks: small-world networks (Watts and Strogatz 1998) and preferential-attachment networks (Barabási

A

		Alters		
		(G,G)	(G, S1)	(S1, S2)
	G	1	1	1
Ego	S1	0	0	0
	S2	0	0	0
	S3	0	0	R

B

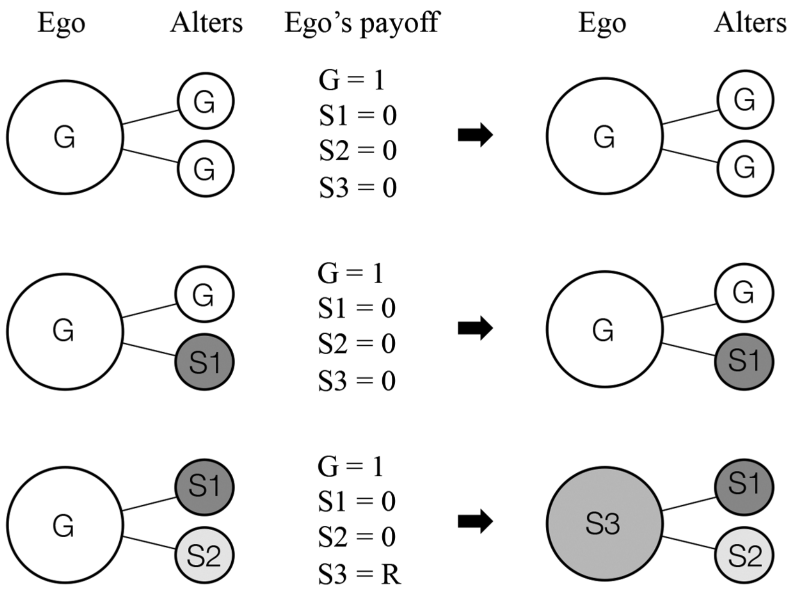


Figure 2. Payoff Structure and Specialization Development with Two Neighbors and Three Items
Note: The outcomes are indicated by the color of the nodes. Colors correspond to the type of specialization (dark gray is for S1, light gray is for S2, medium gray is for S3). White signifies generalization (i.e., lack of specialization). The specialization types S1, S2, and S3 are commutative in the payoff structure.

and Albert 1999). The two models have been used to examine the network effects on human coordination (Centola 2010; Centola and Macy 2007; Kearns et al. 2006; McCubbins et al. 2009; Shirado and Christakis 2017).

The small-world network model allows us to investigate how bridging ties affect the specialization process while holding constant other parameters such as network size, density, chromatic number, and chromatic constraint. In small-world networks, the average shortest path length, that is, the average minimum number of connections that separate nodes in a network, can vary considerably without corresponding changes in density, size of the network, chromatic number, or chromatic constraint. Thus, small-world networks allow us to disentangle the role of the bridging ties that are created in the rewiring process from these other network properties.

Using the preferential-attachment model allows us to explore new hypotheses by examining the effects of item number and chromatic constraint on the emergence of

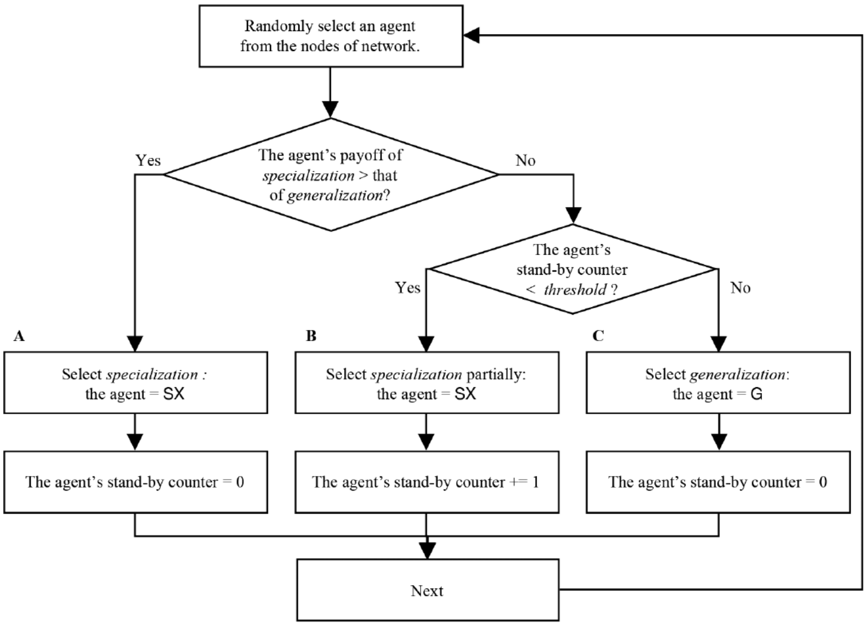


Figure 3. Flow Chart of Graph-Coloring Game in Agent-Based Simulations with Storage Threshold

complementary specialization. Comparable small-world networks have little variance in chromatic constraint, as demonstrated in Figure 1, but the number of complete solutions to the graph-coloring game (i.e., when all agents differentiate themselves from their neighbors) varies widely across different preferential-attachment networks with the same network size and density (Shirado and Christakis 2017). Additionally, in a graph-coloring game, networks generated by the preferential-attachment model are always solvable with a certain number of colors (Kearns et al. 2006). Thus, the preferential-attachment model allows us to examine the effect of different numbers of specializations on the division of labor and thereby investigate the importance of the chromatic number. This step is particularly important because the number of ways labor may be divided via specialization is independent from the topological features that set the chromatic number. Thus, although these are abstract models of possible configurations of social relations, they have important attributes that allow us to explore some of our central hypotheses.

Finally, we consider an observed network to evaluate a more realistic solution space. Data for this network are drawn from Schwimmer's 1960s study of the Orokaiva, a community in Papua New Guinea; the data record the exchange of Taro, a tuber that was the central foodstuff of the Orokaiva, between 22 families (Hage and Harary 1983; Schwimmer 1970).[2] Exchange of cooked Taro was extremely common between the Orokaiva, and Schwimmer interpreted this to indicate ties of intimacy. We use the Orokaiva network because it presents an example of a largely undifferentiated network that could evolve into a division of labor over time. Finally, we compare results across the different types of networks for more insight into how network structure may affect the development of complementary exchange.

RESULTS FOR SMALL-WORLD NETWORKS

First, we consider small-world networks and the effect of bridging ties. Small-world networks are highly clustered networks with low average path length. The combination of clustering and reachability is created by the addition of a low proportion of random connections. These random connections typically reach across the dense, local clusters to link otherwise distant nodes. In doing so, they can dramatically decrease the time it takes for information or various forms of contagion to spread through the network (Watts 1999; Watts and Strogatz 1998). The random connections that span across small-world networks are conceptually related to weak ties (Granovetter 1973), although we refer to them here as bridging ties.

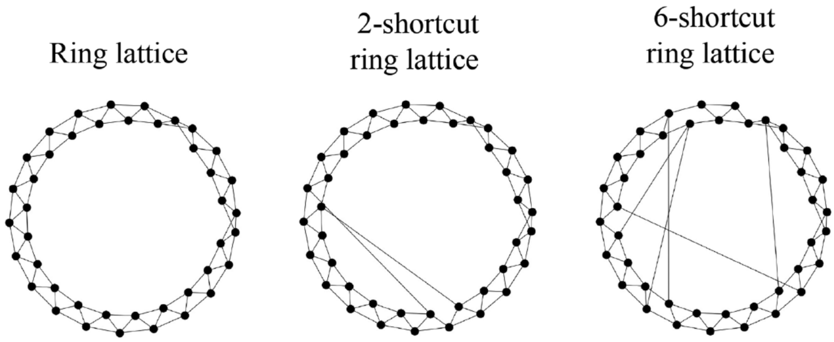
Bridging ties are particularly interesting in terms of evaluating the emergence of the division of labor because of their association with the onset of modernity. As noted previously, closely-bounded community and kin ties are associated with pre-industrial societies, and arm's-length ties are associated with modern market expansion, suggesting an increase in specialization and the spread of the division of labor (Tönnies [1887] 2002). Based on these loose historical generalizations, one might expect bridging ties to encourage the division of labor. Our findings, however, do not support this expectation.

Figure 4 presents descriptive statistics for the small-world network simulations. All the networks have 42 nodes and 84 edges. The density of the networks does not vary because shortcuts are created by repositioning existing links (i.e., rewiring) (Watts and Strogatz 1998). For all networks, each node has a degree of 4, a chromatic number of 3, and a chromatic solution number of 6. We use three items (i.e., equal to the chromatic number) in the small-world network simulations. Small-worlds are generated by the random rewiring of ties, when existing ties in the network are randomly assigned to new nodes. This process also produces some variation in the clustering coefficient and shortest path length across the different realizations of the network type.[3] These types are simulated 500 times. Because there is no variation in the structure of the ring-lattice network, we do not report the standard deviation for it. The clustering coefficient and average shortest path length both decrease with higher rates of rewiring.

Figure 5 presents the results for networks when the number of specializations is varied. The number of specializations can stand in for the idea of different but complementary goods (e.g., food, water, and shelter), the division of tasks in a productive activity (e.g., building a microscope), or the division of complementary areas of expertise within organizations. In the ring lattice, the simplest and most constrained network structure, the number of specializations acts as a strict threshold, where anything less than or equal to the chromatic number leads to the spread of the division of labor to the entire population, and anything greater completely inhibits the spread of complementary specialization. Similarly, the chromatic number provides another inflection point in the two-shortcut and six-shortcut ring-lattice networks, although the difference to either side is less stark, and the variation within the networks with a number of specializations equal to the chromatic number is large. Because nearly all the variance occurs in the networks using a number of specializations determined by the chromatic number, we now explore the structural determinants of that variance.

Figure 6 compares results of the simulation between the small-world networks without and with storage capacity. Here, the number of specializations is equal to the chromatic number of networks (i.e., the number of specializations = 3). Panel A presents results for the condition in which nodes have no storage capacity, and panel B presents results for the condition in which nodes have storage capacity. In panel A, it is clear that the addition of bridging ties through rewiring is associated with differences in the extent of specialization. However, the direction of the association does not follow that suggested by prior theory. The networks with the highest rate of rewiring also have the lowest proportion of specialization, and even a small amount of rewiring shifts the average proportion of specializing agents down by nearly 25 percent.

This result can be understood by considering the way simple network structures can serve as a social guide to agents. The ring lattice is a simple model with a chromatic



	Ring lattice	2-shortcut ring lattic	6-shortcut ring lattice
Number of shortcuts …………	0	2	6
Rewiring probability …………	.000	.024	.071
Number of nodes ……………..	42	42	42
Number of edges ……………..	84	84	84
Density ……………………….	.098	.098	.098
Clustering coefficient ………..	.500	.469	.425
	(n/a)	(.012)	(.021)
Average shortest path length …	5.634	4.994	4.229
	(n/a)	(.353)	(.350)
Chromatic number ……………	3	3	3
Chromatic solution number …..	6	6	6

Figure 4. Descriptive Statistics for Small-World Networks

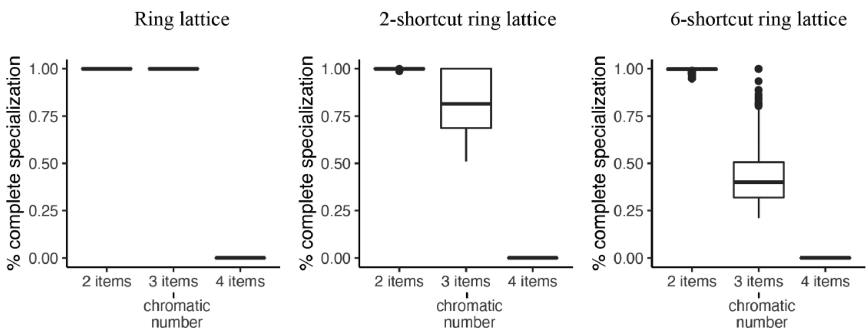


Figure 5. Small-World Network Simulation Results across Various Number of Specializations, without Storage

number of 3 and a solution number of 6. This low number indicates high chromatic constraint. However, the patterned nature of the connections in the network makes choosing a solution at the local level trivial for agents. As a result, there is a 100 percent solution rate. In contrast, adding random connections *decreases* the proportion of agents specializing. Adding two bridging ties that span the network decreases the average path length and

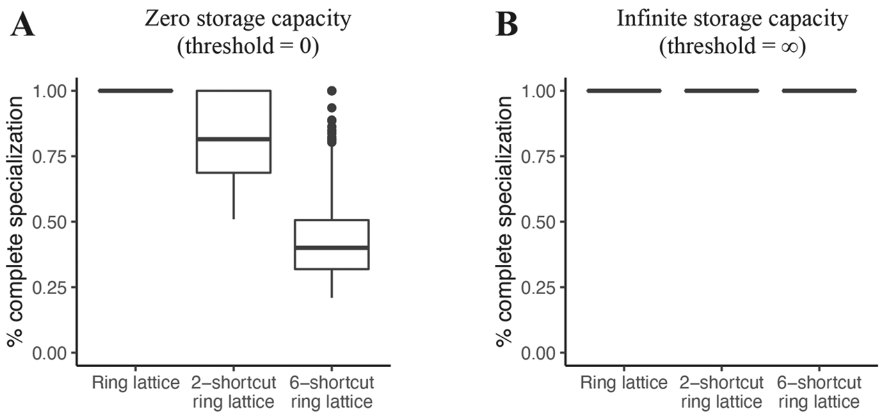


Figure 6. Results of Tie Rewiring and Strategy Capacity in Small-World Network Simulations

also decreases the fraction of the population who are able to choose a specialization that increases their payoff. This decrease is the result of the additional complexity introduced by connections that disrupt the otherwise clear lattice pattern of on-and-off-again specialization choices. Adding additional bridging ties, that is, increasing the likelihood of rewiring, further decreases the proportion of the population that chooses to specialize.

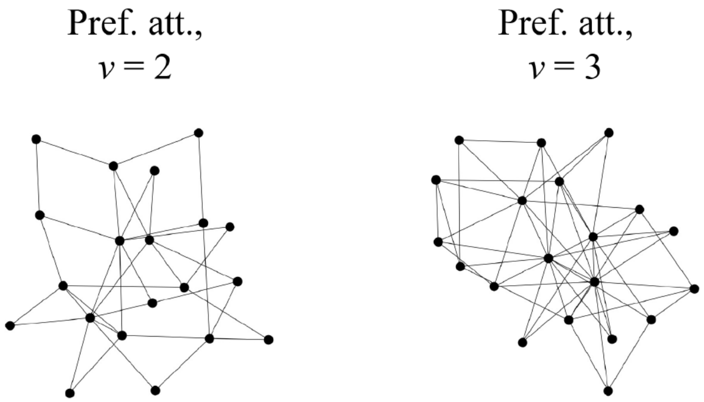
When agents can store goods between rounds, all individuals in each of the three networks are able to find a specialization that resolves the complementarity problem (Figure 6B). In small-world networks, the addition of property allows everyone in the population to successfully engage in complementary specialization whether or not the network contains bridging ties. As reported in the descriptive statistics, these changes in the proportion of the population that specializes are not related to differences in the density of the network, the size of the network, or chromatic constraint.

RESULTS FOR PREFERENTIAL-ATTACHMENT NETWORKS

The effect of chromatic constraint on the possibility of the division of labor is difficult to gauge in small-world networks because its variation is constrained by the low levels of randomization introduced by the rewiring of lattice networks. To explore the effect of chromatic constraint on complementary specialization, we turn to another commonly-used network type: preferential-attachment models.

Preferential-attachment networks capture a common social phenomenon defined by the likelihood of having a connection being conditioned on the number of existing connections. Examples include instances in which having many friends makes it more likely that an individual will make new friends, or when having a higher number of followers on social media increases the likelihood a person will attract new followers. This tendency produces networks with uneven degree distributions. To construct our networks, we used the Barabási-Albert model (Barabási and Albert 1999) where v is a parameter that maps nodal degree to the likelihood of adding a new connection. The Barabási-Albert model differs from the small-world model in that the network is generated by adding new nodes and ties, rather than rewiring existing ties between a fixed set of nodes. One result of this different generative process is greater variation in chromatic constraint.

Figure 7 presents descriptive statistics for the preferential-attachment networks. In Figure 7, v represents the number of edges to attach in the preferential-attachment procedure. We restricted our analysis to networks



	Pref. att., $v = 2$	Pref. att., $v = 3$
Number of edges to attach …….	2	3
Number of nodes ……………..	20	20
Number of edges ……………..	36	51
Density ……………………….	.189	.268
Clustering coefficient ………..	.322	.393
	(.110)	(.075)
Average shortest path length ….	2.156	1.866
	(.072)	(.036)
Chromatic number ……………	3	4
Chromatic solution number …..	421.5	43805.0
	(507.3)	(58075.8)

Figure 7. Descriptive Statistics for Preferential-Attachment Networks

of size 20 because the chromatic polynomial is computationally intensive to calculate, particularly for large networks. The rate of attachment affects the number of edges and therefore the density, so $v = 3$ networks have higher density than $v = 2$ ones. They also have a higher chromatic number. For a solution to exist, nodes in networks set at $v = k$ must have at least $k + 1$ specializations to choose from (Kearns et al. 2006). The clustering coefficient is slightly higher for networks with higher preferential-attachment rates, and the average shortest path length is slightly lower. The average number of solutions is much higher at higher rates of preferential attachment—in the order of 100 times higher. The standard deviation for the number of solutions is also extremely high; it is larger than the average in both network types.

Figure 8 presents results for the simulation at two different levels of attachment for networks operating in the condition of no property when we vary the number of specializations. The results again show the

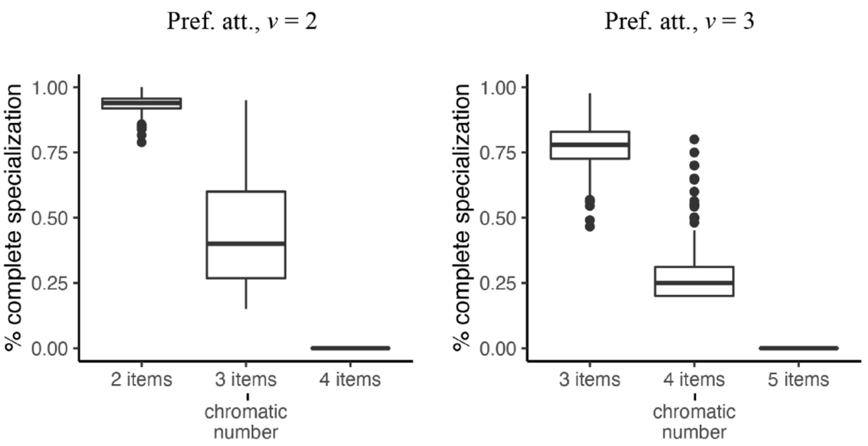


Figure 8. Preferential-Attachment Network Simulation Results across Various Number of Specializations, without Storage

chromatic number is an inflection point for the emergence of the division of labor. If the task is divided into fewer specializations than the chromatic number of the network, most actors can enjoy the benefit of specialization. If the task is divided into a number of specializations greater than the chromatic number, no actor is able to achieve the decentralized coordination necessary for complementary specialization. As before, we focus on the networks using the number of specializations determined by the chromatic number, as this is where most of the variance occurs.

In Figure 9, the *x*-axis represents the chromatic constraint of the networks with logarithmic conversion. The further right that observations fall, the larger the number of solutions they have to resolve the coordination problem. The *y*-axis is the fraction of the total population of nodes that adopt specialization by the end of the simulation. The *r* values indicate the Pearson's product moment correlation coefficient between the logarithmic constraint values and the fraction of specialists.

Comparing the two graphs provides some initial information. Note that the *x*-axis is shifted right for the $v = 3$ network observations. Observations along this dimension begin about where they end for the $v = 2$ networks. As is evident in the descriptive statistics, the chromatic constraint is lower in the denser networks with higher rates of preferential attachment. These networks are less successful at transitioning to specialization: across all observation clusters, a smaller fraction of the population specializes than in the $v = 2$ network.

The same general pattern is repeated within the attachment types. For networks with $v = 2$, having lower chromatic constraint produces lower proportions of complementary specialization. A smaller proportion of the population adopts specialization when the number of solutions is higher. Similarly, for networks with $v = 3$, larger proportions of the population transition to specialization when chromatic constraint is higher. In both, higher chromatic constraint makes it more likely agents will specialize.

Thus, in the condition without storage, chromatic constraint has a clear relationship to the emergence and spread of complementary specialization; however, that association runs in an unexpected direction when we consider only the restrictive side of how constraint can operate. Having more individual latitude for specialization choices might make solving the complementary coordination problem easier; yet, we find that as chromatic constraint increases, the proportion of nodes that successfully specialize decreases. This association cannot be explained by increased density, as density is held constant across the networks with different levels of chromatic

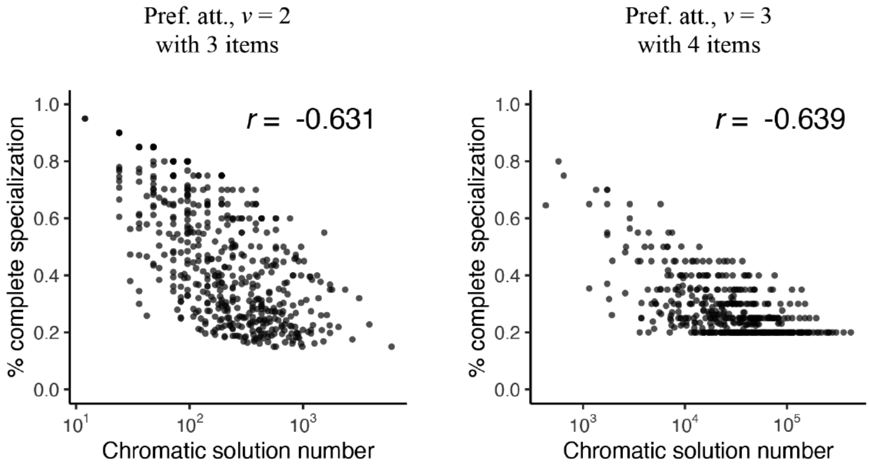


Figure 9. Fraction of Completion by Solution Constraint without Storage in Preferential-Attachment Network Simulations

constraint. It can, however, be explained by a breakdown in the way the structure of the network serves as a social guide for agents. Without this guide, a proliferation of possible solutions leads to local conditions in which agents fail to specialize in the specific way that will make it possible for others to also benefit from specialization. The appropriate specialization choice for the larger network cannot be seen from agents' local positions: the lack of constraint leads them to get lost in a forest of possible pathways.

These findings are consistent with the existence of storage capacity. Figure 10 presents results for preferential-attachment networks that exist under the condition in which property can be stored for later rounds of exchange. The two panels show networks of moderate and higher levels of preferential attachment. In preferential-attachment networks, property again significantly increases the number of agents able to engage in complementary specialization. In the $v = 2$ network, upward of 60 percent of agents are able to specialize when property storage is added, and upward of 50 percent are able to specialize with property storage in the $v = 3$ network. The possibility of storing property encourages the division of labor in these models. Note, however, that not all agents are able to successfully resolve the coordination problem as in the small-world networks. In addition, higher levels of chromatic constraint again encourage the division of labor, especially when property storage is limited.

RESULTS FOR A REAL-WORLD SOCIAL NETWORK

Finally, we examine the effect of chromatic constraint and storage capacity on the emergence of complementary specialization using a real-world social network of gift-exchange among 22 households in a Papuan village (Hage and Harary 1983). The chromatic number for this network is 3. It is decentralized with little variance in the degree distribution. For context, note that Papuan society has strong conceptions of property, and the accumulation and redistribution of resources is central to their system of social status and internal governance (Strathern 1971), but Schwimmer (1970) believed there was a pattern of generalized exchange in the taro network.

To compare how chromatic constraint and storage of property interact with each other in this observed network, we constructed two additional comparison networks based on the first by randomly removing six ties in one case and randomly adding six ties in the other. We chose the number of ties to minimize our manipulation of the network

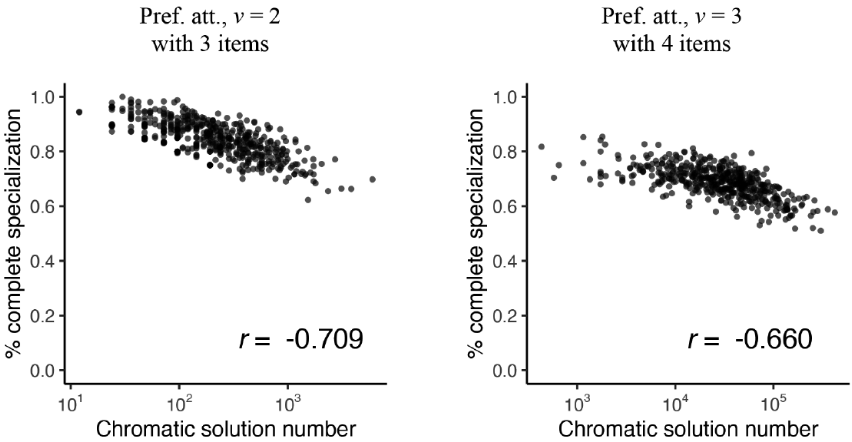


Figure 10. Fraction of Completion with Storage Capacity in Preferential-Attachment Network Simulations

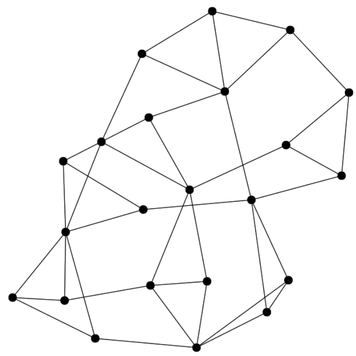
while maximizing variance in chromatic constraint. We used only connected networks for the analysis. Figure 11 presents the relevant descriptive statistics for the network. As expected, there are small changes in density, the clustering coefficient, and path length. The average number of solutions (i.e., the chromatic constraint) varies widely, from 4.5 to 8,188, with only this slight change in network topology. Note that after the addition and removal of the random ties, some networks' chromatic number was no longer 3, but we calculated the number of solutions with three colors for comparison.

The simulation for the real-world network follows a similar process to that outlined in Figure 3. Three connected nodes are chosen at random 500 times as initial seeds. In the modified networks, six dyads are randomly chosen for deletion or insertion in each trial. The observation is the average of 500 replications. Figure 12 illustrates the simulation results using the two extreme levels of the threshold representing storage capacity: 0 and infinitely great. We again see that chromatic constraint and property storage both have a significant effect on specialization dynamics. As shown with the preferential-attachment networks, higher chromatic constraint facilitates complementary specialization and gives more agents the benefit of the division of labor in both the existence and absence of storage capacity. The results provide another indication that chromatic constraint acts as a guide to successful division of labor. In addition, the largest difference in the fraction of the population that successfully specializes occurs between the world with storage and the world without. Even the worst performing network configuration in the stored-property world has a specialization rate above 75 percent, whereas the best performing network configuration in the world without storage barely reaches 25 percent. Storage capacity significantly improves the development of complementary specialization. We further confirmed that adding and subtracting different numbers of ties produced the same pattern.[4]

COMPLEMENTARY SPECIALIZATION AND SOCIAL GUIDES IN THE MODEL RESULTS

Because it is difficult to intuit the specific sequence of actions within the model that produces these results, we now explore the results in the context of simple networks of six nodes without storage capacity. Figure 13 illustrates how chromatic constraint *facilitates* the development of complementary

Original topology of
a real-world social network



	Original network - 6 edges	Original network	Original network + 6 edges
Number of nodes	22	22	22
Number of edges	33	39	45
Density	.143	.169	.195
Clustering coefficient	.271	.339	.310
	(.071)	(n/a)	(.030)
Average shortest path length	2.884	2.493	2.237
	(.138)	(n/a)	(.040)
Chromatic number*	3	3	3
Chromatic solution number ..	8188	72	4.5
	(2674)	(n/a)	(8.0)

* We use the same number of specialization for comparison although some rewired networks have different chromatic number.

Figure 11. Descriptive Statistics for a Real-World Social Network and Its Variations

specialization. Panel A illustrates two different networks of six nodes. Networks α and β both have a chromatic number of 3. Network α has a chromatic solution number of 6. Network β has one additional tie that halves the solution number of the network (i.e., increases chromatic constraint). In the original graph-coloring game, it is harder to find a solution that optimizes coordination in Network β than in Network α because Network β has fewer solutions (Shirado and Christakis 2017).

In each of these simplified simulations, we begin the process with a small triad of specialized nodes. In the example of Figure 13A, when the triad has specialized, the other three agents (Agents *a*, *b*, and *c*) have two possible ways to successfully specialize in Network α, whereas they have only one combination in Network β. This difference is reflected in the solution number of each network.

Figure 13B represents the possible stages of complementary specialization. With this division of labor model, Network α, with a

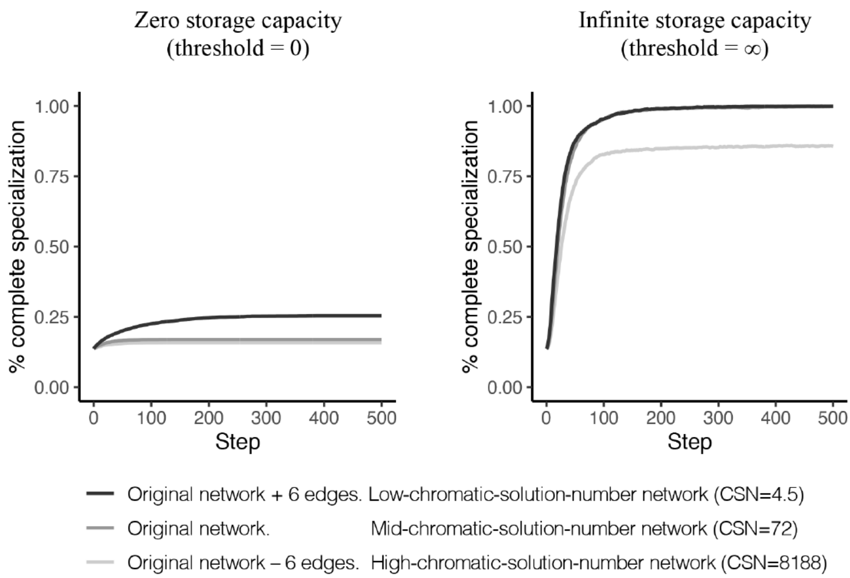


Figure 12. Different Specialization Dynamics with Different Levels of Chromatic Constraint Based on a Real-World Social Network

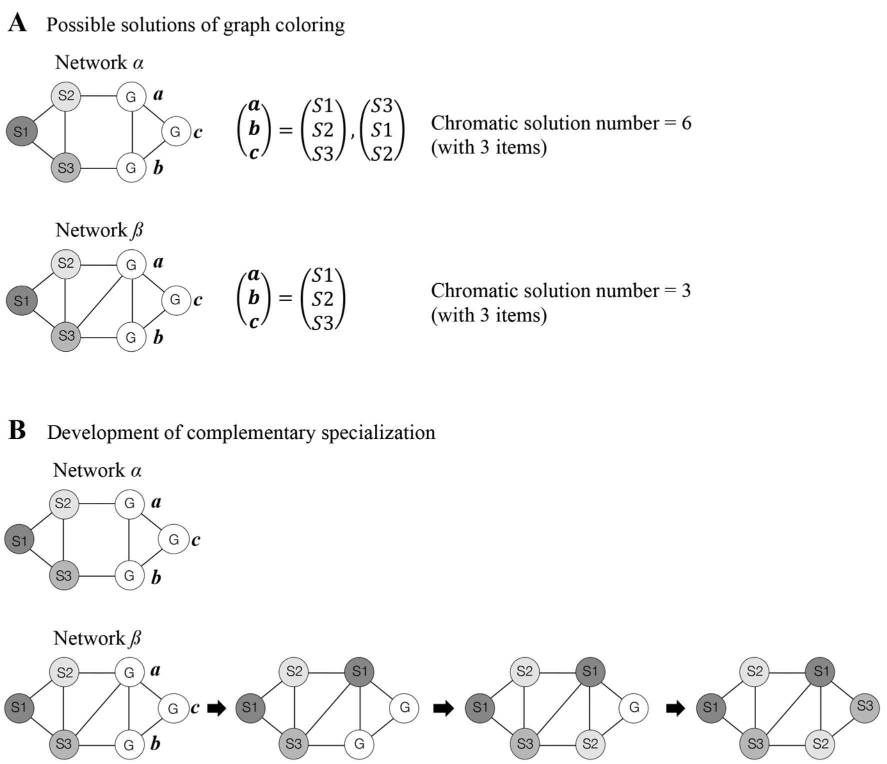


Figure 13. Possible Outcomes for Simple Networks of Six Nodes

solution number of 6, has no further iterations because the generalist agents have no incentives to specialize. Agents *a* and *b* are each connected to a specialist, but they are only connected to one specialist. Thus, if Agent *a* or *b* choose to specialize, they will not be able to procure one of the goods they require at that time. There is thus no incentive to specialize, and the division of labor fails to spread through the network.

The additional tie in Network *β* creates a coordination opportunity that produces a different collective outcome. In this network, Agent *a* is connected to two specialists producing different goods. The benefit to specialization is clear, and Agent *a* chooses its color (in this example, S1). The specialization of Agent *a* creates a situation in which Agent *b* is now tied to two specialists of different goods. Once again, there is an immediate benefit to specialization for the agent. The same process then repeats for Agent *c*. In fact, if one imagined this network of six as one chain in a circular lattice network, it follows that specialization would spread throughout the network as agents' local decisions produce a cascade of solutions. In this case, the constrained choice set given to agents encourages the spread of the division of labor. The network with the higher chromatic constraint, Network *β*, has higher rates of specialization—even though there are fewer global optimal solutions in graph-coloring. With the division of labor model, the constraints imposed by the network structure serve as a social guide to agents that encourages complementary specialization.

CONCLUSIONS

The division of labor is of long-standing theoretical interest because of its relationship to trade and economic growth. The sociological literature generally agrees that the division of labor phenomenon encompasses a broad set of interdependence problems, not limited to sustenance (Gibbs and Poston 1975; Kemper 1972). The continued existence of these problems of interdependency provides good reason for continuing to think through the emergence of the division of labor using a theoretical and formal lens. It is a fascinating problem of interdependent, decentralized coordination. Despite the relevance of network structure to economic processes, previous researchers have not applied formal models and the tools of network science to the division of labor problem. We believe this may be because they were not able to find the right model. Multipartite extensions of network analysis and the graph-coloring model make it possible to explore the formal dimensions of complementary specialization, an investigation we pursued here.

Before interpreting the results, it is important to ask whether this modified graph-coloring game captures enough of the complexities of real-world exchange to serve as a useful model for the evolution of a division of labor. The division of labor game is a stylized, conceptual model that ignores many factors, such as the uneven distribution of resources and skills (Shirado, Iosifidis, and Christakis 2019). In this sense, the game is a low-dimensional agent-based model appropriate for the exploration and clarification of existing theories and the generation of new potential mechanisms (Bruch and Atwell 2013), as is our intention here. In particular, we expect the model will shed light on national, regional, and micro-level processes of complementary specialization. In these cases, both relational ties and infrastructural elements, like river networks, railway networks, and air traffic patterns, will likely provide varying levels of chromatic constraint that may have significant effects on specialization patterns—and thereby economic development rates (Tóth et al. 2021). We believe insights from the model can also be applied to the division of productive activity within organizations, which have informal and formal constraints on interaction patterns. The model cannot, however, be generalized to all types of specialization; in particular, our results would not apply to specialization based on difference rather than complementarity, or the competitive specialization theorized in the organizational ecology literature. However, the method we use, the modification and application of the graph-coloring problem, can also be used to explore these other processes, including specialization based on difference.

In particular, we note that our approach should have applications for the literature on diversity, problem-solving, and organizational performance. The benefits to diversity are well documented (see Aral and Van Alstyne 2011; Gomez and Lazer 2019; Page 2008). Diversity is different than the division of labor mainly in that it may require less strict coordination between actors. In contrast to the division of labor, each actor does not necessarily need access to a full set of diverse values (if those are countable) to enjoy the benefits of diversity. On the other hand, diversity can also be considered through the lens of interdependent specialization. For diversity to exist, actors need to be different from each other (i.e., local redundancy should be avoided). Thus, some principles of the graph-coloring problem do apply. Our investigation provides a model for thinking through structural issues related to increasing diversity as well as other types of specialization based on difference. Because the nature of the interdependence varies, the results will differ from what we found here, but such investigations should provide a much broader base for understanding all types of specialization. The application of graph-coloring and multipartite network analysis can provide an interesting and fruitful path forward for the investigation of many different social-coordination processes and problems.

Our findings are consistent with previous work suggesting network structure is an important factor in predicting whether individuals within groups will be able to achieve the transformation into a division of labor. However, early hypotheses from classical authors—some of the last to propose structural theories about the emergence of the division of labor—do not receive much support. Using the small-world network model enabled us to investigate the role of bridging ties in encouraging the division of labor. We found that in the condition without storage capacity, random connections across the network hampered the spread of complementary specialization, and they made no difference in the condition with property.

Although we did not directly test Durkheim's theory about the importance of increasing social density, we did hold density constant in both the small-world network and the two types of preferential-attachment networks. The proportion of agents that chose to specialize and the rate at which complementary specialization permeated the network varied dramatically, suggesting density cannot be the only cause of variation in the development of the division of labor. Future studies should test the role of density more directly.

We found that chromatic constraint plays an important and heretofore unknown role in promoting complementary specialization. Chromatic constraint may be difficult to observe with the naked eye and is not yet commonly available as a measure in popular network analysis software, but it is important because it provides a sense of the solution space for coordination attempts concealed within network topology (Shirado and Christakis 2017). The chromatic number, that is, the fewest colors needed in graph-coloring of a focal network, indicates there is a limit to the number of specializations actors can take on and still successfully engage in complementary coordination within a network. If actors need fewer specializations than the chromatic number, most of them can achieve the division of labor under most of the structural conditions we have explored. If actors need more specializations than the chromatic number, they are less likely to achieve the division of labor. If actors need the same number of specializations as the chromatic number, the development and benefit of specialization vary greatly depending on the chromatic constraint and network type.

In statistical analysis of the association between chromatic constraint and the fraction of the population that specializes in preferential advantage networks (see the Appendix), the solution number is significant even when controlling for the clustering coefficient and average shortest path length ($p < .001$ for each preferential-attachment model with linear regression; $N = 200$) and has a larger effect size than the other two measures. The independent effect of chromatic constraint on the emergence of the division of labor suggests this network feature likely plays an important and as yet unexamined role in other social processes—particularly coordinative and cooperative activities between groups. It may be difficult to observe without prior knowledge of its relevance, but knowing of its significance makes it much easier to identify. The results indicate that structurally-derived constraint can serve as a social guide to successful coordination and the division of labor.

We also found that storage capacity had an extremely consequential effect on the development of complementary specialization. In all the networks we analyzed, storage dramatically increased the proportion of specializing agents, uniformly raising the specializing population to 100 percent for the lattice, 2-shortcut, and 6-shortcut networks. Storage is of larger theoretical importance because it is linked to norms and laws regulating the possession of private property. Storage can indicate technologies of preservation, or that private property rights allow individuals to exclude other actors from access to and use of their goods, thereby preserving goods for future rounds of exchange. Sociologists often focus on the disadvantages of private property, in particular its relationship to inequality (Carruthers and Ariovich 2004). Private property allows for the accumulation of resources, which makes inequality possible. And the manipulation of property rights can institutionalize disadvantages for subpopulations, such as when women are denied the right to own land. In contrast, economists often emphasize the importance of private property rights for economic development (Acemoglu, Johnson, and Robinson 2001, 2002; De Soto 2000; Hall and Jones 1999; Knack and Keefer 1995; North 1990, 1994; Rodrik, Subramanian, and Trebbi 2004). This perspective largely identifies property rights as increasing individuals' incentives to engage in productive activity. In our model, storage of property does not change the incentive structure for agents, but it still has an effect. These results suggest a significant benefit of private property may be the way it can help solve the problem of coordinating complementary interdependence in groups. The effect of property also highlights the importance of the temporal ordering of exchange relations. Sequentiality and simultaneity, proxied in storage capacity or property rights, operate very differently under similar structural conditions (Erikson 2018).

Altogether, the results suggest some broad heuristics. Complementary specialization can be difficult to achieve in a decentralized fashion unless the network structure tightly constrains, and thereby guides, individual behavior. This finding suggests centralized authorities, like states and managers, may have been important in developing the complex patterns of complementary specialization we find in developed economies. Repetitive, tightly constrained networks, like the ring-lattice network, are more likely to produce complementary specialization in decentralized settings than are networks with increased randomness and complexity, indicating that structural constraint can encourage the division of labor. Fine-tuning the pattern of connections between actors has the potential to improve the likelihood of developing the division of labor. Property can also help with the coordination problem in decentralized settings. Finally, dividing labor into too many specializations may make decentralized coordination difficult.

Future research will require additional modeling, experimental tests, and large-scale data collection efforts. We saw that different types of networks (i.e., small-world or preferential-attachment networks) had different proportions of agents choose specialization. Computational models can expand the range of predictions for different types of networks, but these should also be tested empirically. One could conduct laboratory tests in which network connections are manipulated to vary chromatic constraint and task division is recorded. In addition, different systems of exchange, such as the Kula Ring, could be identified so their chromatic constraint can be measured alongside the extent of the division of labor, and the association between the two could be tested. To this end, the empirical literature from the twentieth century on measuring the division of labor (Clemente and Sturgis 1972; Gibbs and Browning 1966; Gibbs and Poston 1975) could be revived and applied to instances in which regular patterns of exchange can be ascertained through an examination of infrastructure (e.g., highways) (Perz et al. 2013) and transportation patterns (e.g., air traffic, import-export flows). Interaction patterns in organizations and the cultivation of areas of expertise may prove fruitful areas for empirical investigation—even the way friendship networks affect medicinal herb cultivation in remote locations (Díaz-Reviriego et al. 2016) could be used to test the model developed here. Appropriate time frames can be ascertained by measuring the stability of these patterns within different settings.

Future work should also consider models and empirical tests of the importance of chromatic constraint in more dynamic networks, in which actors can change their exchange partners. We were unable to do this here because it was necessary to hold structure constant to gauge the effect of the various features of network topology (Shirado, Iosifidis, and Christakis 2019; Shirado et al. 2019). We are particularly interested in exploring evolutionary dynamics related to the emergence of different market and exchange patterns, such as the gendered division of labor or regional specialization patterns. For example, perhaps the distinctive pattern of the Kula Ring evolved as a way to facilitate the division of labor between these communities. Dynamic models will be necessary to explore these evolutionary processes.

APPENDIX

A. Without storage (threshold = 0).

	Pref. att., v=2 with 3 items		Pref. att., v=3 with 4 items	
	Estimated coefficient	p value	Estimated coefficient	p value
Intercept	0.446	< 0.001	0.280	< 0.001
Chromatic solution number (log)	-0.133	< 0.001	-0.068	< 0.001
Clustering coefficient	-0.022	0.033	-0.003	0.610
Average shortest path length	-0.043	< 0.001	-0.002	0.679

B. With storage (threshold = ∞).

	Pref. att., v=2 with 3 items		Pref. att., v=3 with 4 items	
	Estimated coefficient	p value	Estimated coefficient	p value
Intercept	0.845	< 0.001	0.687	< 0.001
Chromatic solution number (log)	-0.041	< 0.001	-0.039	< 0.001
Clustering coefficient	0.017	< 0.001	0.002	0.407
Average shortest path length	0.018	< 0.001	0.008	< 0.001

Figure A1. Results of Linear Regression Analysis on the Specialization Outcomes of Preferential-Attachment Networks
Note: All covariates are normalized for comparison.

Acknowledgments

The authors are grateful for valuable comments received at the Penn State workshop on cultural and political sociology, the Network Ecology conference held at Stanford University, and the Center for Empirical Research on Stratification and Inequality at Yale University. We owe a special debt of gratitude to the reviewers and editors, William Goetzmann, and Daniel DellaPosta. The order of authors is alphabetical. The authors contributed equally to the development of the manuscript.

Data and Code Availability

The data and code in this manuscript are available at Mendeley Data (http://dx.doi.org/10.17632/235bt6swkg.1). The simulation code was written in Python. The code for analysis and visualization was written in R.

Notes

1. For robustness, we also ran models in which generalization and specialization were randomly assigned across nodes at the initial state, and a model in which specialization was irreversible. Results are available from the authors upon request.
2. Constraints on processing time for the calculation of the chromatic polynomial—even for only the chromatic number—make the analysis of much larger networks a lengthy process.
3. To further clarify, we control the variation across the simulated small-world networks so the solution constraint for all networks is 6. Without this control, the solution constraint would vary, and in some cases no full solution would be possible. We also restrict the set of networks to those with the exact number of shortcuts (2 or 6), which is different from a typical graph generator of small-world networks using rewiring probabilities (Watts and Strogatz 1998).
4. Results are available from the authors upon request.

ORCID iDs

Emily Erikson https://orcid.org/0000-0003-2092-4676

Hirokazu Shirado https://orcid.org/0000-0003-4545-7859

References

Abbott, Andrew. 1988. "Transcending General Linear Reality." *Sociological Theory* 6(2):169–86.

Acemoglu, Daron, Simon Johnson, and James A. Robinson. 2001. "The Colonial Origins of Comparative Development: An Empirical Investigation." *The American Economic Review* 91(5):1369–401.

Acemoglu, Daron, Simon Johnson, and James A. Robinson. 2002. "Reversal of Fortune: Geography and Institutions in the Making of the Modern World Income Distribution." *The Quarterly Journal of Economics* 117(4):1231–94.

Aral, Sinan, and Marshall Van Alstyne. 2011. "The Diversity-Bandwidth Trade-Off." *American Journal of Sociology* 117(1):90–171.

Barabási, Albert-László, and Réka Albert. 1999. "Emergence of Scaling in Random Networks." *Science* 286(5439):509–12.

Bearman, Peter. 1997. "Generalized Exchange." *American Journal of Sociology* 102(5):1383–415.

Becker, Gary S., and Kevin M. Murphy. 1992. "The Division of Labor, Coordination Costs, and Knowledge." *The Quarterly Journal of Economics* 107(4):1137–60.

Bell, Nancy E., and Barry M. Staw. 1989. "People as Sculptors versus Sculpture: The Roles of Personality and Personal Control in Organizations." Pp. 232–51 in *Handbook of Career Theory*, edited by M. B. Arthur, D. T. Hall, and B. S. Lawrence. Cambridge, UK: Cambridge University Press.

Boorman, Scott A. 1974. "Island Models for Takeover by a Social Trait Facing a Frequency-Dependent Selection Barrier in a Mendelian Population." *Proceedings of the National Academy of Sciences of the United States of America* 71(5):2103–7.

Bruch, Elizabeth E., and John Atwell. 2013. "Agent-Based Models in Empirical Social Research." *Sociological Methods & Research* 44(2):186–221.

Bruch, Elizabeth E., and Robert D. Mare. 2006. "Neighborhood Choice and Neighborhood Change." *American Journal of Sociology* 112(3):667–709 (https://doi.org/10.1086/507856).

Burt, Ronald S. 2004. "Structural Holes and Good Ideas." *American Journal of Sociology* 110(2):349–99.

Carroll, Glenn R. 1987. *Publish and Perish*. Greenwich, CT: Jai Press.

Carroll, Joseph. 1995. "Evolution and Literary Theory." *Human Nature* 6(2):119–34.

Carruthers, Bruce G., and Laura Ariovich. 2004. "The Sociology of Property Rights." *Annual Review of Sociology* 30(1):23–46.

Centola, Damon. 2010. "The Spread of Behavior in an Online Social Network Experiment." *Science* 329(5996):1194–7.

Centola, Damon, and Michael Macy. 2007. "Complex Contagions and the Weakness of Long Ties." *American Journal of Sociology* 113(3):702–34.

Chaudhuri, Kamalika, Fan Chung Graham, and Mohammad S. Jamall. 2008. "A Network Coloring Game." In *Internet and Network Economics*, edited by C. Papadimitriou and S. Zhang. WINE 2008. *Lecture Notes in Computer Science*, Vol. 5385. Berlin: Springer (https://doi.org/10.1007/978-3-540-92185-1_58).

Clemente, Frank, and Richard B. Sturgis. 1972. "The Division of Labor in America: An Ecological Analysis." *Social Forces* 51(2):176–82.

Cohen, Lisa E. 2013. "Assembling Jobs: A Model of How Tasks Are Bundled into and across Jobs." *Organization Science* 24(2):432–54.

Coleman, James S. 1994. *Foundations of Social Theory*. Cambridge, MA: Harvard University Press.

De Soto, Hernando. 2000. *The Mystery of Capital: Why Capitalism Triumphs in the West and Fails Everywhere Else*. New York: Basic Books.

Diamond, Peter A. 1982. "Aggregate Demand Management in Search Equilibrium." *American Journal of Sociology* 90(5):881–94.

Díaz-Reviriego, Isabel, Lara González-Segura, Álvaro Fernández-Llamazares, Patricia L. Howard, José Luis Molina, and Victoria Reyes-García. 2016. "Social Organization Influences the Exchange and Species Richness of Medicinal Plants in Amazonian Homegardens." *Ecology and Society* 21(1) (http://dx.doi.org/10.5751/ES-07944-210101).

Durkheim, Emile. [1893] 1996.*The Division of Labor in Society*. New York: The Free Press.

Erikson, Emily. 2018. "How Group Events Can Shape Network Processes." *Sociological Theory* 36(2):187–93.

Feld, Scott L. 1981. "The Focused Organization of Social Ties." *American Journal of Sociology* 86(5):1015–35.

Foley, Duncan. 2019. "Social Coordination Problems in Classical and Marxian Political Economy." *Review of Social Economy* 78(1):4–34.

Garey, Michael R., and David S. Johnson. 1979. *Computers and Intractability*. New York City: W. H. Freeman and Company.

Gibbs, Jack P. 2003. "A Formal Restatement of Durkheim's 'Division of Labor' Theory." *Sociological Theory* 21(2):103–27.

Gibbs, Jack P., and Harley L. Browning. 1966. "The Division of Labor, Technology, and the Organization of Production in Twelve Countries." *American Sociological Review* 31(1):81–92.

Gibbs, Jack P., and Dudley L. Poston. 1975. "The Division of Labor: Conceptualization and Related Measures." *Social Forces* 53(3):468–76.

Gomez, Charles J., and David M. J. Lazer. 2019. "Clustering Knowledge and Dispersing Abilities Enhances Collective Problem Solving in a Network." *Nature Communications* 10 (https://doi.org/10.1038/s41467-019-12650-3).

Graeber, David. 2011. *Debt: The First 5,000 Years*. Brooklyn, NY: Melville House.

Granovetter, Mark S. 1973. "The Strength of Weak Ties." *American Journal of Sociology* 78(6):1360–80.

Granovetter, Mark S. 1978. "Threshold Models of Collective Behavior." *American Journal of Sociology* 83(6):1420–43.

Hage, Per, and Frank Harary. 1983. *Structural Models in Anthropology*. Cambridge, UK: Cambridge University Press.

Hall, Robert E., and Charles I. Jones. 1999. "Why Do Some Countries Produce So Much More Output Per Worker Than Others?" *The Quarterly Journal of Economics* 114(1):83–116.

Hannan, Michael T., and John Freeman. 1977. "The Population Ecology of Organizations." *American Journal of Sociology* 82(5):929–64.

Hannan, Michael T., László Pólos, and Glenn R. Carroll. 2007. *Logics of Organization Theory*. Princeton, NJ: Princeton University Press.

Hechter, Michael. 1978. "Group Formation and the Cultural Division of Labor." *American Journal of Sociology* 84(2):293–318.

Hofbauer, Josef, and Karl Sigmund. 1998. *Evolutionary Games and Population Dynamics*. Cambridge, UK: Cambridge University Press.

Jensen, Tommy R., and Bjarne Toft. 2011. *Graph Coloring Problems*. Hoboken, NJ: John Wiley & Sons.

Kapsalakis, Lauren. 2011. "The Informal and Formal Economies of Indigeneity among the Toba of Barrio Nam Qom, Formosa, Argentina." Dissertation in Anthropology, University of Pennsylvania.

Kearns, Michael, Siddharth Suri, and Nick Montfort. 2006. "An Experimental Study of the Coloring Problem on Human Subject Networks." *Science* 313(5788):824–27.

Kemper, Theodore D. 1972. "The Division of Labor: A Post-Durkheimian Analytical View." *American Sociological Review* 37(6):739–53.

Kitts, James A. 2006. "Collective Action, Rival Incentives, and the Emergence of Antisocial Norms." *American Sociological Review* 71(2):235–59 (https://doi.org/10.1177/000312240607100204).

Knack, Stephen, and Philip Keefer. 1995. "Institutions and Economic Performance: Cross-Country Tests Using Alternative Institutional Measures." *Economics and Politics* 7(3):207–27.

Lawrence, Paul R., and J. W. Lorsch. 1967. *Organization and Environment*. Boston, MA: Harvard Business School Press.

Lu, Hui, Charlene Rohr, Marco Hafner, and Anna Knack. 2018. "China Belt and Road Initiative: Measuring the Impact of Improving Transportation Connectivity on Trade in the Region." Santa Monica, CA: RAND Corporation (https://www.rand.org/pubs/research_reports/RR2625.html).

Mackie, Gerry. 1996. "Ending Footbinding and Infibulation: A Convention Account." *American Sociological Review* 61(6):999–1017.

Macy, Michael W., and Robert Willer. 2002. "From Factors to Actors: Computational Sociology and Agent-Based Modeling." *Annual Review of Sociology* 28(1):143–66.

Malinowski, Bronislaw. 1984. *Argonauts of the Western Pacific*. Prospect Heights, IL: Waveland Press.

McCubbins, Mathew D., Ramamohan Paturi, and Nicholas Weller. 2009. "Connected Coordination." *American Politics Research* 37(5):899–920.

North, Douglass C. 1990. *Institutions, Institutional Change and Economic Performance*. Cambridge, UK: Cambridge University Press.

North, Douglass C. 1994. "Economic Performance through Time." *The American Economic Review* 84(3):359–68.

Ostrom, Elinor. 1997. "Private and Common Property Rights." Pp. 332–79 in *Encyclopedia of Law and Economics*, edited by G. De Geest. Cheltenham, UK: Edward Elgar Publishing.

Page, Scott E. 2008. *The Difference: How the Power of Diversity Creates Better Groups, Firms, Schools, and Societies*. Princeton, NJ: Princeton University Press.

Perrow, Charles. 1986. "Economic Theories of Organization." *Theory and Society* 15(1/2):11–45.

Perz, Stephen G., Martha Rosero, Flavia L. Leite, Lucas Araujo Carvalho, Jorge Castillo, and Carlos Vaca Mejia. 2013. "Regional Integration and Household Resilience: Infrastructure Connectivity and Livelihood Diversity in the Southwestern Amazon." *Human Ecology* 41(4):497–511.

Polanyi, Karl. 1944. *The Great Transformation*. Boston, MA: Beacon Press.

Puranam, Phanish, Oliver Alexy, and Markus Reitzig. 2014. "What's 'New' about New Forms of Organizing?" *Academy of Management Review* 39(2):162–80.

Raveendran, Marlo, Luciana Silvestri, and Ranjay Gulati. 2020. "The Role of Interdependence in the Micro-Foundations of Organization Design: Task, Goal, and Knowledge Interdependence." *Academy of Management Annals* 14(2):828–68.

Reeve, Andrew. 1986. *Property*. Atlantic Highlands, NJ: Humanities Press.

Rodríguez-Clare, Andrés. 1996. "Multinationals, Linkages, and Economic Development." *The American Economic Review* 86(4):852–73.

Rodrik, Dani, Arvind Subramanian, and Francesco Trebbi. 2004. "Institutions Rule: The Primacy of Institutions over Geography and Integration in Economic Development." *Journal of Economic Growth* 9(2):131–65.

Schieffelin, Edward L. 1981. "Evangelical Rhetoric and the Transformation of Traditional Culture in Papua New Guinea." *Comparative Studies in Society and History* 23(1):150–56.

Schwimmer, Erik G. 1970. *Exchange in the Social Structure of the Orokaiva*. New York City: St Martin's Press.

Shelton, Beth A., and Daphne John. 1996. "The Division of Household Labor." *Annual Review of Sociology* 22:299–322.

Shirado, Hirokazu, and Nicholas A. Christakis. 2017. "Locally Noisy Autonomous Agents Improve Global Human Coordination in Network Experiments." *Nature* 545(7654):370–74.

Shirado, Hirokazu, George Iosifidis, and Nicholas A. Christakis. 2019. "Assortative Mixing and Resource Inequality Enhance Collective Welfare in Sharing Networks." *Proceedings of the National Academy of Sciences of the United States of America* 116(45):22442–44.

Shirado, Hirokazu, George Iosifidis, Leandros Tassiulas, and Nicholas A. Christakis. 2019. "Resource Sharing in Technologically Defined Social Networks." *Nature Communications* 10 (https://doi.org/10.1038/s41467-019-08935-2).

Simon, James G., and Herbert A. March. 1958. *Organizations*. Cambridge, MA: Blackwell.

Smith, Adam. [1776] 2003. *An Inquiry into the Nature and Causes of the Wealth of Nations*. New York: Bantam Dell.

Smith, John M. 1982. *Evolution and the Theory of Games*. Cambridge, UK: Cambridge University Press.

Stark, David. 2011. *The Sense of Dissonance: Accounts of Worth in Economic Life*. Princeton, NJ: Princeton University Press.

Strathern, Andrew. 1971. *The Rope of Moka: Big-Men and Ceremonial Exchange in Mount Hagen New Guinea*. Cambridge, UK: Cambridge University Press.

Strauss, Anselm. 1985. "Work and the Division of Labor." *The Sociological Quarterly* 26(1):1–19.

Thompson, James. 1967. *Organizations in Action*. New York: Routledge.

Tönnies, Ferdinand. [1887] 2002. *Community and Society*. Mineola, NY: Dover Publications.

Tóth, Gergő, Johannes Wachs, Riccardo Di Clemente, Ákos Jakobi, Bence Ságvári, János Kertész, and Balázs Lengyel. 2021. "Inequality Is Rising Where Social Network Segregation Interacts with Urban Topology." *Nature Communications* 12 (https://doi.org/10.1038/s41467-021-21465-0).

Watts, Duncan J. 1999. "Networks, Dynamics, and the Small-World Phenomenon." *American Journal of Sociology* 105(2):493–527.

Watts, Duncan J., and S. H. Strogatz. 1998. "Collective Dynamics of 'Small-World' Networks." *Nature* 393(6684):440–42.

Weick, Karl. 1969. *The Psychology of Organizing*. Boston, MA: Addison-Wesley.

White, Harrison C. 1963. *An Anatomy of Kinship: Mathematical Models for Structures of Cumulated Roles*. Englewood Cliffs, NJ: Prentice-Hall.

Wilmers, Nathan. 2020. "Job Turf or Variety: Task Structure as a Source of Organizational Inequality." *Administrative Science Quarterly* 65(4):1018–57.

Wrzesniewski, Amy, and Jane E. Dutton. 2001. "Crafting a Job: Revisioning Employees as Active Crafters of Their Work." *The Academy of Management Review* 26(2):179–201.

Emily Erikson is Associate Professor of sociology and the school of management (by courtesy) and the Joseph C. Fox Academic Director of the Fox International Fellowship. She uses comparative historical and computational methods to study economic growth, institutional development, and social coordination in global contexts. Her recent book, *Trade and Nation: How Companies and Politics Reshaped Economic Thought* (Columbia University Press 2021) argues that the rise of the company and merchant political marginalization encouraged the development of classical economic thought.

Hirokazu Shirado is an Assistant Professor of the Human-Computer Interaction Institute in the School of Computer Science at Carnegie Mellon University. His research focuses on the role of social interactions and technology in the emergence of social order and in the collective confrontation of social dilemmas. He is particularly committed to the experimental study of cooperative behaviors as they manifest through interactions between people located within social networks. He is also studying hybrid systems of humans and machines, and how the addition of artificial agents can help humans address challenges of collective action in social networks.